A Guard-Band Strategy for Managing False-Accept Risk

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Abstract

When performing a calibration, the risk of incorrectly declaring a device as in-tolerance (false-accept risk) is dependent upon several factors. Those factors include the specified tolerance limit, guard-band, the calibration process uncertainty and the a priori probability that the device is intolerance. A good estimate of the a priori probability may be difficult to obtain. Historical or device population information for estimating the a priori probability may not be readily available and may not represent the specific device under test.

A common strategy for managing measurement decision risk is to choose a guard-band that results in the desired false-accept risk given the tolerance limit, the calibration process uncertainty and the a priori probability. This paper presents a guard-band strategy for managing false-accept risk with only limited knowledge of the a priori probability that a device is intolerance.

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When determining if measurement quantities are within specified tolerances, ANSI/NCSLI Z540.3-2006 specifies that the maximum level of false-accept risk be no more than 2%. False-accept risk is the probability that measuring an out-of-tolerance device will indicate an intolerance condition due to measurement error. False-reject risk is the probability that measuring an in-tolerance device will indicate an out-of-tolerance condition due to measurement error. False-accept and false-reject occurrences have financial consequences, and therefore, minimizing both is often a worthwhile objective.

One strategy for managing false-accept risk is to apply a guard-band such that the acceptance limits are more stringent that the tolerance limits. A common practice (see [3]) is to set the guard-band to a value equal to the 95% expanded uncertainty of the calibration process. This level of guard-band guarantees the Z540.3 false-accept risk requirement and is attractive in that it only requires information that many calibration organizations routinely manage (that is, the tolerance limits and the 95% expanded uncertainty, which is set as the guard-band). However, when using a guard-band to reduce false-accept risk, a corresponding increase results in the false-reject risk. With the guard-band set to the 95% expanded uncertainty, the false-reject risk can be disproportionately high (see Figure 3).

An alternative to applying a guard-band equal to the 95% expanded uncertainty is to determine the false-accept risk and set an appropriate guard-band, if necessary, that adjusts the false-accept risk to the desired level. To determine the level of false-accept (or false-reject risk) for a calibration measurement, the following information is necessary:

- Tolerance limits
- Guard-band
- Calibration process uncertainty
- A reasonable estimate of the a priori probability that a device is in-tolerance

The a priori probability is the likelihood that a device is in-tolerance prior to performing the calibration. It is typical to estimate the a priori probability from the observed in-tolerance rate for a population of like devices. However, if historical observations are unavailable, or if there is reason to believe the device that is the subject of calibration does not belong to the observed population, other means of estimating the a priori probability are necessary.

Managing the estimate of the a priori probability requires additional effort compared with defining a guard-band equal to the 95% expanded uncertainty. This paper presents a guard-band strategy to meet the Z540.3 false-accept requirement that does not require significant knowledge of the a priori probability and yet, achieves a reasonable false-reject risk.

Determining False-Accept and False-Reject Risk

False-accept risk can be determined by evaluating the joint probability density function that models a calibration measurement (see [1]). Assuming Gaussian distributions for the calibration process uncertainty and the a priori probability, the joint probability density function is:

$$p(e_{dut}, y) = p_0\left(e_{dut}\right) p_m\left(y - e_{dut}\right) = \frac{1}{\sqrt{2\pi}\sigma_0} e^{-\frac{(e_{dut})^2}{2\sigma_0{}^2}} \cdot \frac{1}{\sqrt{2\pi}\sigma_m} e^{-\frac{(y-e_{dut})^2}{2\sigma_m{}^2}}$$

where

e_{dut} = the error of the device under test, which the calibration attempts to quantify

y = the observed calibration result

σ_0 = standard deviation of the a priori probability distribution

σ_m = standard deviation of the measurement error (standard uncertainty)

The joint probability density function defines probability over a two-dimensional surface area. The total probability for a given two-dimensional rectangular area is found by integrating the joint probability density function over a region. That is, the probability for a given region is:

$$P_R = \iint_R p_0(e_{dut}) p_m(y - e_{dut}) dA$$ Equation 1

where R defines a particular region. To determine false-accept risk, assuming symmetrical two-sided tolerances, it is necessary to evaluate Equation 1 over two regions defined as:

$$T \le e_{dut} \le \infty \text{and} - A \le y \le A$$

and,

$$-\infty \le e_{dut} \le \ -T \text{and} \ -A \le y \le A$$

where

T = tolerance limit

A = acceptance limit

and the acceptance limit is defined as the difference between the tolerance limit and the guardband:

$$A = T - GB$$

Likewise, to determine false-reject risk, it is necessary to evaluate Equation 1 over the regions defined as,

$$-T \le e_{dut} \le T \text{and} -\infty \le y \le -A$$

and,

$$-T \le e_{dut} \le T \text{and} A \le y \le \infty$$

False-Accept Characteristics

To evaluate Equation 1, it is necessary to estimate the standard deviation for the a priori probability distribution. Assuming a Gaussian distribution, the standard deviation can be estimated as:

$$\sigma_0 = \frac{T}{F^{-1}\left(\frac{1+p}{2}\right)}$$ Equation 2

where

T = tolerance limit

p = observed in-tolerance probability

F^{-1} = inverse normal distribution function

From Equations 1 and 2, it is possible to generate a data set containing false-accept risk as a function of in-tolerance probability and TUR [1]. Figure 1 illustrates a data set for which the acceptance and tolerance limits are equal.

1. Test uncertainty ratio, as defined in paragraph 3.11 of [2].

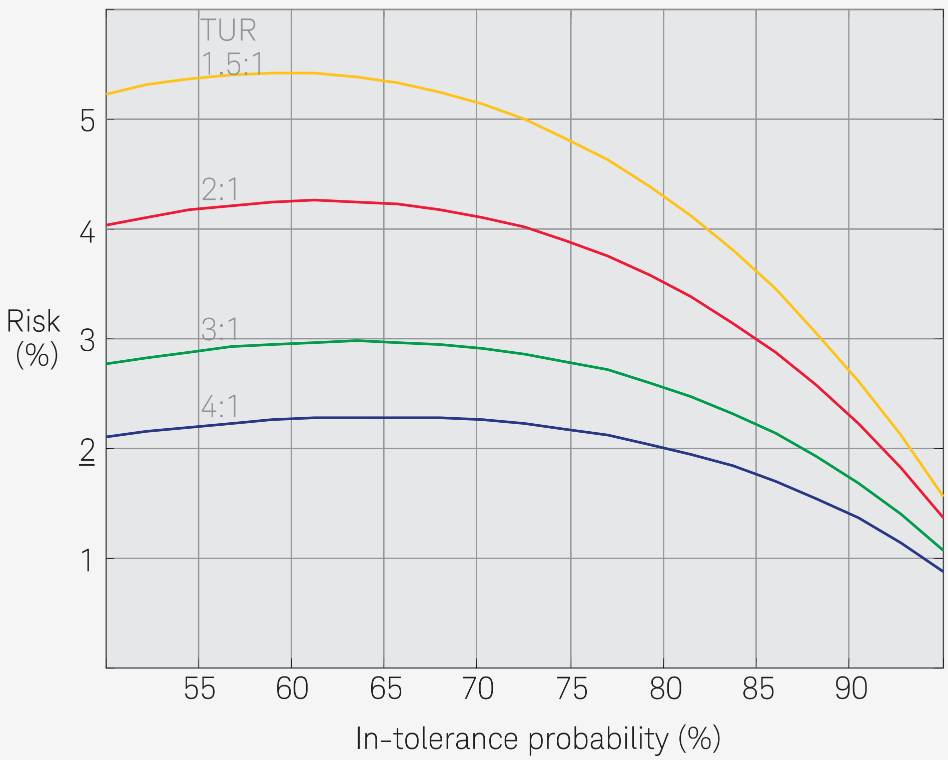

Figure 1. False-accept risk (where the acceptance limits equal the tolerance limits)

As can be observed, for all values of TUR, false-accept risk decreases as the in-tolerance probability approaches 100%. To understand this, imagine a population of devices. Recall that a false-accept is to randomly select a device that happens to be out-of-tolerance but appears to be in-tolerance due to measurement error. If all devices are in-tolerance, then no out-of-tolerance devices exist within the population for which a false-accept is possible, so the probability of false-accept approaches 0%.

Interestingly, as the in-tolerance probability approaches 0%, the false-accept risk also decreases. Consider that as the in-tolerance probability decreases, the device population spreads beyond the tolerance limits. As it spreads, there comes a point that the majority of devices are now out-oftolerance and the number of devices near the tolerance limits decrease. Eventually, randomly selecting a device close enough to the tolerance limits that it might appear as in-tolerance due to measurement error becomes unlikely.

Given that false-accept risk approaches 0% at the extreme ends of the in-tolerance probability range, the maximum false-accept risk exists at an intermediate in-tolerance probability level for a given TUR.

Applying a guard-band to a calibration measurement (that is, setting the acceptance limits tighter than the tolerance limits) reduces false-accept risk. Applying a guard-band in this fashion has the effect of lowering the risk curves shown in Figure 1. For a given TUR, it is possible to apply just enough guard-band so that the maximum risk level is below a desired level. For Z540.3 compliance, the maximum level is 2%. Applying guard-band to manage the maximum possible false-accept risk, referred to as managed risk guard-band, assures compliance for any level of intolerance probability.

Managed Risk Guard-band

Applying guard-band to manage maximum false-accept risk results in a guard-band that is always less than the 95% expanded uncertainty. Accordingly, the acceptance limits can be expressed as follows:

$$A = T - U_{95\%} \times M \qquad \text{Equation 3}$$

where

A = acceptance limit

T = tolerance limit

$U_{95\%}$ = calibration process 95% expanded uncertainty

M = multiplier: the fraction of the 95% expanded uncertainty for which the acceptance limits provide the desired false-accept risk

Using Equation 3 to define the acceptance limits and setting the risk equation to equal the Z540.3 required 2% false-accept risk:

$$2\% = \iint_R p_0(e_{dut}) p_m(y - e_{dut}) dA$$

It is possible to solve for M. The maximum false-accept point for a given TUR can be found visually from Figure 2, or alternatively, by using numerical search algorithms (see Appendix). Solving for M at the maximum false-accept risk points guarantees false-accept risk is always below a specified level for a given TUR. Table 1 shows values of M at in-tolerance probabilities corresponding to the maximum false-accept risk as a function of TUR.

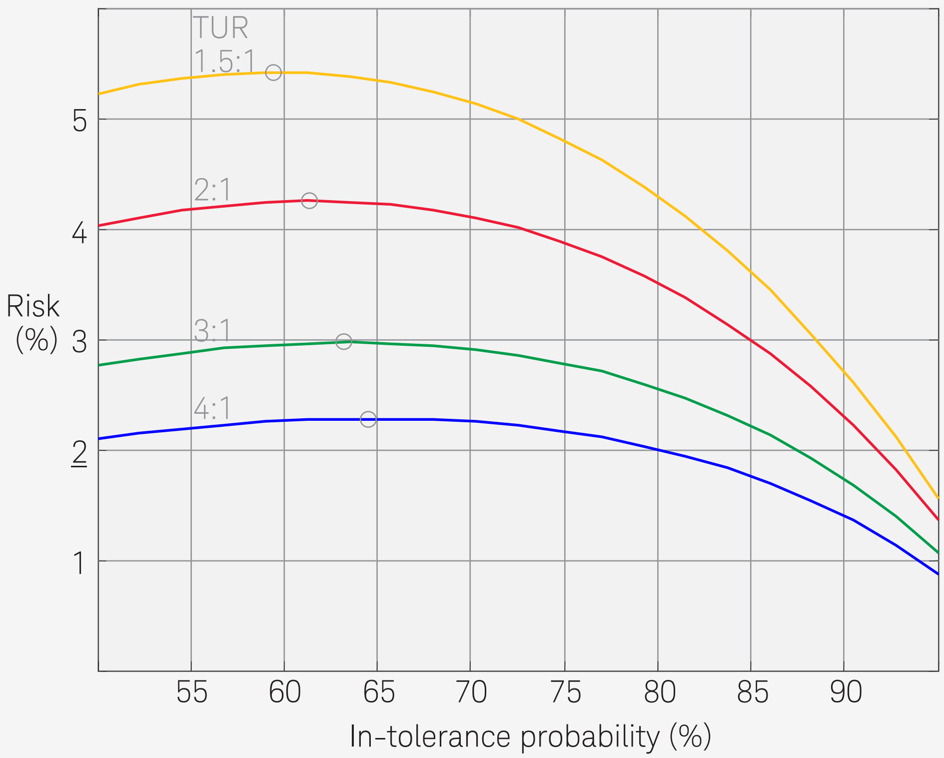

Figure 2. Maximum false-accept risk

Table 1. Managed risk data

TUR	In-tolerance probability for maximum false-accept risk	Maximum false-accept risk	M ×100% (to achieve 2% false-accept risk)
1.5:1	59.62%	5.420%	35.89%
2:1	61.50%	4.249%	27.93%
3:1	63.55%	2.968%	15.36%
4:1	64.65%	2.281%	5.32%

By curve fitting M versus TUR (see Appendix), it is possible to derive an empirical equation for determining M as a function of TUR, denoted $M_{2\%}$ to indicate the equation represents a maximum 2% false-accept risk. That equation[2] is as follows:

$$M_{2\%} = 1.04 - e^{(0.38 \cdot \log(TUR) - 0.54)} \qquad \text{Equation 4}$$

It is possible to use Equation 4 to determine acceptance limits that guarantees the Z540.3 false-accept risk requirement and only requires minimal knowledge of the a priori probability distribution. Specifically, Equation 4 assumes the a priori probability density function is Gaussian and centered within the tolerance limits, but otherwise, it is independent of the distribution spread.

2. The log() function is the natural logarithmic function.

Comparing Guard-Band Strategies

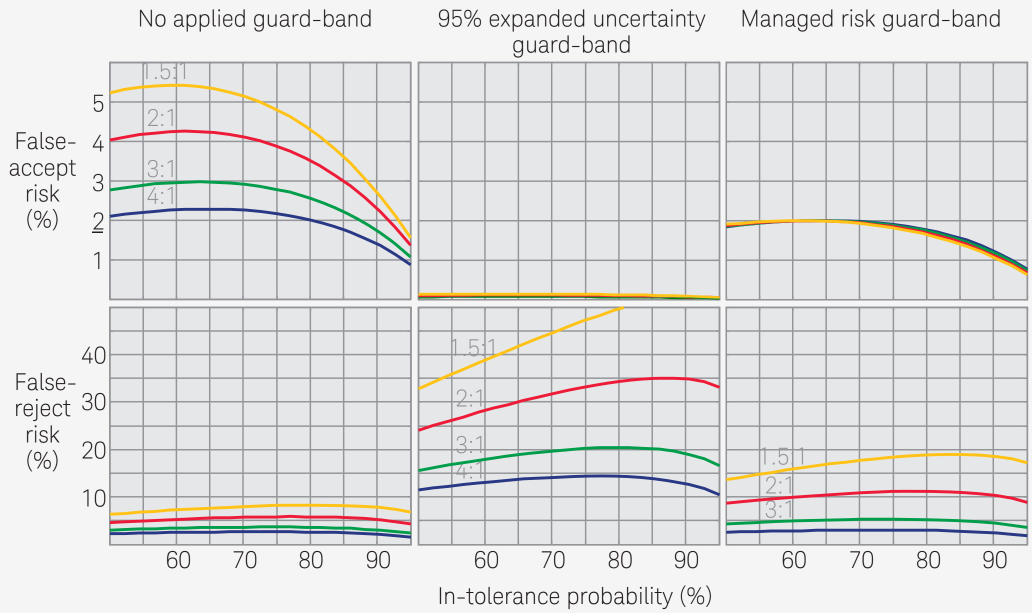

Figure 3. Comparison of guard-band strategies

Figure 3 provides a graphical comparison of different guard-band strategies. The upper-left and lower-left plots show false-accept and false-reject risk without applying guard-band (i.e., the acceptance limits equal the tolerance limits). A guard-band equal to the 95% expanded uncertainty significantly lowers false-accept risk to less than 0.15% for virtually all TUR (upper-middle plot). This more than meets the Z540.3 2% false-accept risk requirement. In this case, however, the false-reject rate can be significant (lower-middle plot). The upper-right and lower-right plots show false-accept and false-reject risk using a managed risk guard-band. For a managed risk guard-band, the acceptance limits are set by combining Equations 3 and 4 as:

$$A_{2\%} = T - U_{95\%} \times \left[1.04 - e^{(0.38 \cdot \log(TUR) - 0.54)} \right] \qquad \text{Equation 5}$$

Choosing acceptance limits using Equation 5 adjusts each TUR false-accept curve so that the maximum false-accept risk is never more than 2%. Compared with a 95% expanded uncertainty guard-band, the impact on false-reject risk is significantly less.

Conclusions

A managed risk guard-band provides a false-accept risk generally between 1% and 2% for most in-tolerance probabilities and TUR. The false-accept risk is reasonably insensitive to TUR and is never more than 2%, and therefore, guarantees the Z540.3 2% risk requirement. A managed risk guard-band ensures Z540.3 risk requirements without requiring knowledge of the standard deviation for the a priori probability distribution, which can be a problematic statistic to obtain and manage. Applying a managed risk guard-band requires virtually the same effort as a 95% expanded uncertainty guard-band; however, the false-reject risk is significantly lower for the managed risk guard-band. Moreover, only false-reject risk is sensitive to TUR. This allows setting quality standards around minimum TUR based primarily on consideration of false-reject risk.

Appendix

Matlab, version 7.3.0.267 (R2006b), was use for all numerical analysis for this paper.

To develop the managed risk guard-band equation, first maximum false-accept risk values for each TUR (listed in Table 2) were determined using the Matlab fminbnd() function. The fminbnd() function finds the minimum of a single-variable function on a fixed interval. The function in this case numerically integrates the joint probability density function using the Matlab dblquad() function. The dblquad() performs a numerical double integration. With values for maximum false-accept risk as a function of TUR, the Matlab fsolve() function was used to find values of M that result in 2% false-accept risk.

Curve fitting the M versus TUR data was a two-step process. The first step involved taking the natural logarithm of both M and TUR and then optimizing a fixed offset added to M for the best linear fit of the transformed data. Using this two-step process provided the best fit as well as a relatively simple equation for M as a function of TUR. The curve fit used all the M and TUR data in Table 2.

Table 2. Extended managed risk data

TUR	In-tolerance probability for maximum false-accept risk	Maximum false-accept risk	M ×100% (2% false-accept risk)
1.1:1	57.15%	6.956%	43.68%
1.2:1	57.89%	6.495%	41.58%
1.3:1	58.54%	6.092%	39.59%
1.5:1	59.62%	5.420%	35.89%
1.75:1	60.67%	4.763%	31.72%
2:1	61.50%	4.249%	27.93%
2.5:1	62.71%	3.495%	21.22%
3:1	63.55%	2.968%	15.36%
3.5:1	64.18%	2.579%	10.11%
4:1	64.65%	2.281%	5.32%
5:1	65.34%	1.852%	–3.23%
6:1	65.80%	1.559%	–10.81%
8:1	66.40%	1.184%	–24.08%
10:1	66.76%	0.955%	–35.73%
12:1	67.01%	0.800%	–46.37%
15:1	67.26%	0.643%	–61.13%
19:1	67.47%	0.510%	–79.49%

Figure 4 shows M versus TUR from Table 2 using 'o' symbols. The continuous line represents the fitted data using Equation 4 displayed as percentage.

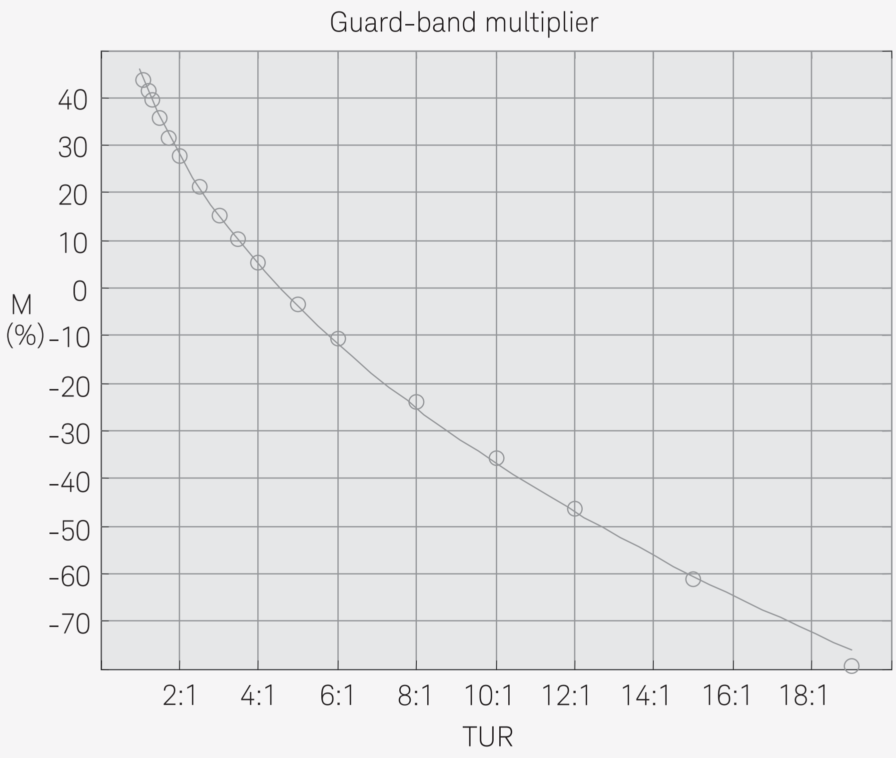

Figure 4. M **versus TUR**

The technique used to generate Equation 4 is not restricted to Gaussian assumptions or the 2% maximum false-accept risk criteria. The same process is suitable for developing similar guard-band equations for other scenarios.

References

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3. ILAC-G8:1996, Guidelines on Assessment and Reporting of Compliance with Specification, International Laboratory Accreditation Cooperation.

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